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EFFECT OF HYDRAULIC CONDUCTIVITY ON THE RESIDENCE TIME OF METEORIC GROUND WATER IN BERMUDIAN- AND BAHAMIAN-TYPE ISLANDS

H. L. Vacher
Department of Geology
University of South Florida
Tampa, Florida 33620

T. O. Bengtsson
Southwest Water Management District
2379 Broad Street
Brooksville, Florida 33512

ABSTRACT

In isolated islands consisting of young limestones, it is common for units with higher hydraulic conductivity (K) to underlie units with lower K. The different hydrostratigraphic units are commonly separated by paleosols or emergence surfaces, and the increasing K with age reflects the more extensive development of secondary permeability in the older units. The contrast in K is commonly one or more orders of magnitude.

In the larger islands of the Bahamas and in Big Pine Key, Florida, fresh-water lenses are large enough that the fresh-water/sea-water interface reaches to the buried high-K units; the lenses in these "Bahamian-like islands" are truncated, with strong geologic control of the base of the lens. In Bermuda, the succession of different-K units is more lateral, reflecting the lateral relative-age pattern resulting from the shoreward accretion of successive interglacial beach-dune complexes during the Pleistocene buildup of that island; lenses in "Bermudian-type islands" are strongly asymmetric, with greatest thicknesses toward the lower-K shoreline. In small islands, such as Holocene oolitic cays of the Bahamas, lenses tend to be symmetric because the islands tend to be composed of homogeneous, single-K sediment. An exception is the asymmetry of atoll-island lenses that are skewed in accordance with a lagoon-to-reef increase in grain size.

The average residence time of ground water in island lenses can be calculated from Dupuit-Ghyben-Herzberg analysis given the areal geometry of the island, distribution and magnitude of K, and distribution and magnitude of recharge. For strip islands with uniform recharge, the calculation is by integration of equations for the water-table elevation and depth to interface (which gives the volume of the lens). Then the average residence time is the volume of fresh water divided by total recharge. For homogeneous-K islands, average residence times are in the range of 5 to 20 years, if K is in the range of 10-100 m/day, porosity is 20-40 percent, recharge is a few tens of cm/yr, and the strip island is 1000 m wide; the result is directly proportional to the width of the island. In Bermudian-type islands, the effect of a strip which is half the width of the island and an order of magnitude more permeable is to reduce the average residence time by about 50 percent from that of the lens in the homogeneous-K island composed only of the low-K unit. In Bahamian-type islands, the effect of the buried high-K unit is not as large. For example, if the contact were at about 30 or 50 percent of the depth of the lens that would be present if only the lower-K unit were present, then the average residence time would be reduced by about 35 and 20 percent, respectively, given an order-of-magnitude contrast of hydraulic conductivity between the superposed layers.

INTRODUCTION

Fresh ground water in small oceanic islands typically occurs in the form of a Ghyben-Herzberg lens (Fig. 1). The typical picture of a Ghyben-Herzberg lens is like that of Figure 1A, where the lens is symmetric and completely contained in a single rock unit. Different geometries occur in Bermuda (Fig. 1B) and the larger islands of the Bahamas (Fig. 1C). These different geometries reflect differences in the spatial variation of hydraulic conductivity in these islands.

The shape and size of an island lens can be calculated given the areal geometry of the island, the distribution and magnitude of recharge, and the distribution and magnitude of hydraulic con-

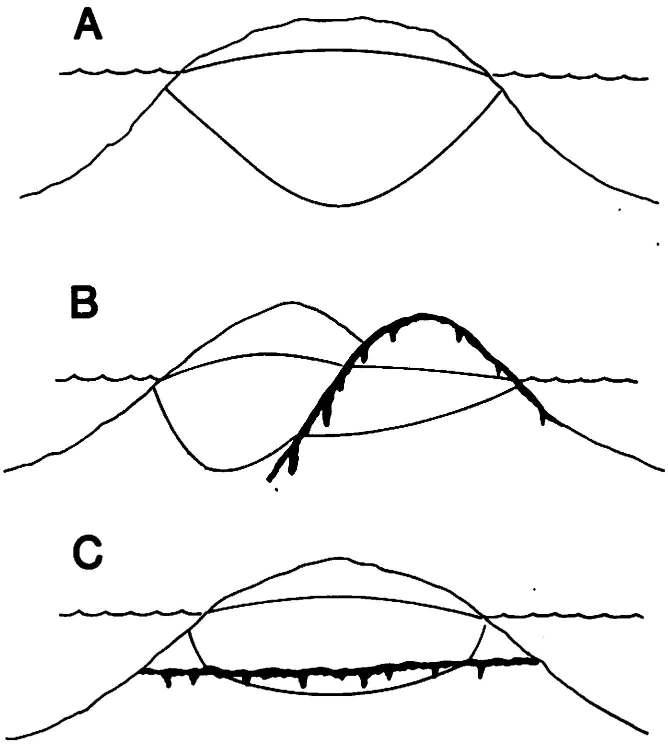


Fig. 1. Three types of island lenses considered in this paper: In A, the island is composed of homogeneous sediment. In B, the island consists of a younger, less permeable unit that onlaps a more permeable limestone. In C, the older, more permeable limestone completely underlies the less permeable unit.

ductivity (Fetter, 1972; Vacher, 1988). Equations for infinite-strip islands with the kinds of hydraulic-conductivity variations shown in Figure 1 are derived by Vacher (1988). The purpose of this paper is to extend the analysis to assess the residence time of fresh ground water in these types of islands.

BACKGROUND

Lenses in General

The fresh ground water is derived from recharge on the island and flows shoreward to discharge at the shoreline. Because of the shoreward flow, the elevation of the water table decreases toward the shoreline in accordance with Darcy's Law. Because of the difference in densities between the fresh ground water and underlying seawater, the column of fresh ground water extends to some depth below sea level. If there is no flow in the salt-water region (i.e., salt-water head is zero), the depth to the freshwater/salt-water interface is related to the elevation of the water table by the well-known Ghyben-Herzberg Principle. This stipulates that the ratio of the interface depth to the water-table elevation at opposite ends of an equipotential (Hubbert, 1940; Vacher, 1988) is equal to the density-difference ratio (the density of the fresh water divided by the difference in densities between the fresh water and seawater). This ratio is usually about 40:1. With horizontal ground-water flow, equipotentials are vertical, so the Ghyben-Herzberg Principle applies along a vertical line. Then, the shape of the interface is simply a 40-fold-exaggerated mirror image of the water table. The water table and interface, therefore, define a lens-shaped geometry, the Ghyben-Herzberg lens.

The residence time of fresh ground water in the lens refers to the length of time for water to flow from its point of entry at the water table to its discharge at the shoreline. As illustrated in Figure 2, this amount of time depends on where the water enters the lens, if recharge is distributed across the island. For example, the residence time of water entering at A is larger than that of water entering at B, and it is largest for water that enters in the vicinity of the

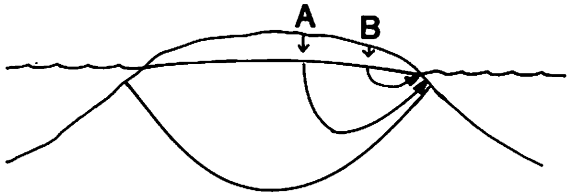


Fig. 2. Island lens with two flow paths indicated. Residence time along A is larger than that along B.

divide. Despite this variability, the overall average is the volume of water in the lens divided by the total flux through it (Vacher et al., in review). For an infinite-strip island,

$$\tau = nV/RL \qquad (1)$$

where is the average residence time, n is porosity, V is the volume of the lens (per unit length parallel to the shoreline), R is recharge, and L is width of the cross section.

The diagrams of Figures 1 and 2 are oversimplifications, because they portray the fresh-water/salt-water contact as a sharp interface. Actually a gradational transition or mixing zone is present (Fig. 3). The presence of such a transition zone implies that there is a discharge of brackish ground water to the shoreline. From the conservation of salt, this implies a circulation of seawater (Cooper, 1959). The inland flow of seawater, in turn, implies that salt-water heads beneath the lens are negative, that is, less than sea level. For such cases of non-zero salt-water heads, the relationship between the depth to the interface and the elevation of the water table is given by the Hubbert equation (Hubbert, 1940; Vacher, 1988). This equation can be considered a two-term, or expanded version of the Ghyben-Herzberg Principle. The negative salt-water heads mean that the ratio of interface depth to water-table elevation is somewhat larger than 40; therefore, the depth to the interface (the midline of the transition zone, in this case) is deeper than that found by not considering the negative salt-water heads. Residence times of fresh ground water would be somewhat larger, because V is larger in equation 1.

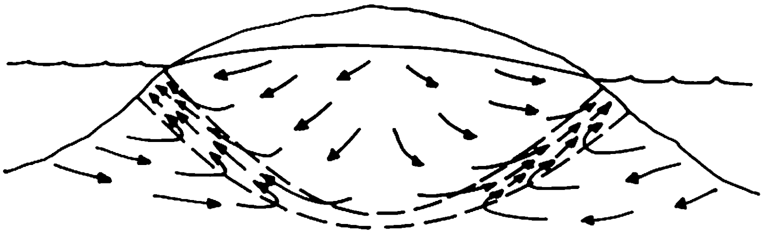


Fig. 3. Diagrammatic flow system in an island lens and associated transition zone.

Method of Analysis

First-cut analyses of Ghyben-Herzberg lenses assume that (1) salt-water head is zero and (2) equipotentials of the fresh-water flow are vertical. The first assumption means that the presence of the transition zone is ignored and that sea level all around the island is at the same elevation; this allows the one-term Ghyben-Herzberg Principle to be used. The second assumption, that flow is horizontal, means that the Ghyben-Herzberg Principle can be applied to equipotentials, rather than curved ones. The justification for this assumption is analyzed by Vacher (1988), and follows from the fact that these lenses are very thin relative to their width -- much more so than the cartoons that portray these ground-water bodies would suggest. The horizontality of the flow system allows for Dupuit analysis, a standard analytical procedure that greatly simplifies application of Darcy's Law to flow problems.

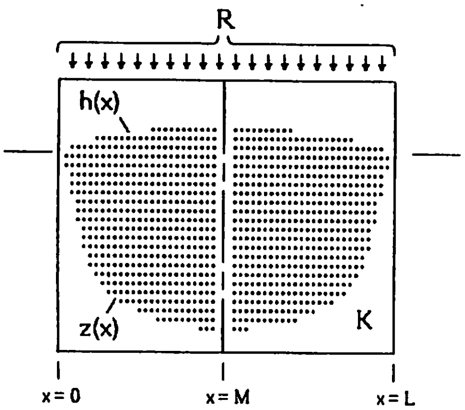
Combination of Dupuit analysis with the one-term Ghyben-Herzberg Principle leads to Dupuit-Ghyben-Herzberg analysis (Bear, 1972; Fetter, 1972; Vacher, 1988), which is used here to evaluate the volume of lenses in islands with the kinds of hydraulic-conductivity variations as occur in Bermuda and the larger Bahamian islands (Fig. 1). From the volume, estimates of residence time follow immediately from equation 1. The results underestimate the average residence time of fresh ground water, because Dupuit-Ghyben-Herzberg analysis ignores the presence of the mixing zone. To take the mixing zone into account would require variable-density solute-transport modeling, which is a much more difficult procedure. It is suspected that the results from this more sophisticated approach, for fresh-water residence times, would not be significantly larger than those calculated here.

It is assumed throughout this paper that the islands are infinite strip, meaning that the bounding shorelines are parallel and the islands are very long relative to their width. It is also assumed that recharge is constant, so that the lenses are steady-state. Because the intent of the paper is to focus on the effects of variations in hydraulic conductivity, recharge is taken also to be areally uniform.

THE BASIC LENS: HOMOGENEOUS HYDRAULIC CONDUCTIVITY

Configuration of the Lens

Mathematically and conceptually, the simplest strip-island lens is one in an island with homogeneous and isotropic hydraulic conductivity (K) and areally uniform recharge (R). The eleva-



R = recharge
K = hydraulic conductivity
L = island width
M = coordinate of divide
x = distance from divide
ρ_f = fresh-water density
ρ_s = seawater density
$\alpha = \rho_f/(\rho_s - \rho_f) \cong 40$

Then:

(1) Elevation of water table above sea level: $h^2 = \frac{R}{K(\alpha+1)}(Lx - x^2)$

(2) Depth of interface below sea level: $z^2 = \frac{R\alpha^2}{K(\alpha+1)}(Lx - x^2)$

(3) Thickness of fresh-water column: $H^2 = \frac{R(\alpha+1)}{K}(Lx - x^2)$

(4) Divide: $M = L/2$

Fig. 4. Parameters and equations for water table, interface and fresh-water thickness as function of distance from shoreline in homogeneous, strip-island lens from Dupuit-Ghyben-Herzberg analysis. Derivations are in Vacher (1988).

tion of the water table, the depth to the interface, and the thickness of the fresh-water column are easily calculated from Dupuit-Ghyben-Herzberg expressions included in Figure 4. As indicated by these equations, the highest elevation of the water table, the largest depth of the interface and thickest column of fresh ground water occur along the central axis of the island. The lens is symmetric about this axis.

Some features of these homogeneous-island solutions are worth noting. First, the thickness/width ratio of the lens is a function of the ratio of recharge to hydraulic conductivity. That is, the thickest column of fresh water (H_m) is given by

$$H_m^2 = R(\alpha + 1)L^2/4K \quad (2),$$

where the terms are as defined in Figure 4. Therefore, the thickness/width ratio of the lens is

$$H_m/L = (\alpha + 1)^{1/2}(R/K)^{1/2}/2 \quad (3).$$

Because R/K is commonly in the range of 10^{-4} to 10^{-6}, this equation implies that island lenses are commonly very thin compared to their width -- on the order of 1:30 to 1:100 (Budd, 1984; Vacher, 1988). Typical pictures of island lenses, where they are drawn as fat, squat icebergs, are really cartoons; real lenses, in fact, are long, thin slivers. As an example, consider an island 1 km across, with K=10 m/day, and R=0.3 m/yr (which represent a fine sand and a Bermudian recharge, respectively). According to equation 2, the thickest column of fresh water is 28.3 m; from equation 3, the thickness/width ratio is about 3 percent. The conclusion to be drawn is that fresh-water, phreatic diagenesis is clearly limited to only a relatively thin zone below sea level in the exposed limestones -- in small islands, at least.

The second point illustrated by equation 3 is the effect of hydraulic conductivity on the thickness of the fresh-water column. Hydraulic conductivity ranges over several orders of magnitude. Islands composed of limestones with relatively large hydraulic conductivity will have relatively thin lenses. For example, if K=90 m/day (corresponding to upper Pleistocene calcarenites of Bermuda), rather than the 10 m/day assumed in our hypothetical example, then the thickness and thickness/width ratio would both be reduced to a third, 9.4 m and about 1 percent, respectively.

The third point is the linear dependence of the thickness on the width of the island (equation 2). If our hypothetical island were 10 km rather than 1 km wide (with K=10 m/day), the thickness of the lens then would be 10 times larger, or 283 m (and the thickness/width ratio would still be 2.8 percent). In real islands, however, this greater depth of the interface in larger islands generally means that the interface reaches down to some older unit that has a different hydraulic conductivity. If this occurs, then hydraulic conductivity is not homogeneous within the fresh-water phreatic zone, and the equations of Figure 4 do not apply. This is exactly the case documented by Cant and Weech (1986) for the larger islands of the Bahamas. These islands are sufficiently large that the interface intersects deeper units that have a significantly larger hydraulic conductivity (e.g., Fig. 1C).

Residence Time

The volume of the strip-island lens with homogeneous K and uniform R is found by integrating Hdx from x=0 to x=L, where H(x) is from Figure 4. Combination of this volume with equation 1 leads to an expression for the average residence time of fresh ground water in strip-island lenses

$$\tau = (n \pi L/8) (\alpha + 1)^{1/2}/(R K) \quad (4).$$

For the hypothetical example where L=1000 m, K=10 m/d, and R=0.3 m/yr, the average residence time works out as 15.2 yrs, if the porosity were 0.2.

Figure 5 shows the dependence of residence time on recharge and hydraulic conductivity. The standard island is the example used above, where the maximum interface depth is 28.3 m and the average residence time is 15.2 yrs. From equation 4, it is seen that the residence time would be half as large if either the recharge or hydraulic conductivity were increased by a factor of 4. The reason for this is obvious in the case of hydraulic conductivity: if K were increased by a factor of 4, the lens would be everywhere thinned by a factor of 2 (Fig. 4), so V/RL also would be decreased by a factor of 2. The dependence on recharge is not as immediately apparent. If the recharge were increased by a factor of 4, the lens would be twice as large, but, because the total flux, RL, would also be four times as large, V/RL would be half as large. Accordingly, the residence time varies with the -1/2 power with both hydraulic conductivity and recharge, as is indicated by equation 4 and shown in Figure 5.

Because residence time varies linearly with respect to both porosity and island size, the residence times in Figure 5 are easily converted to give the residence times in islands with different L and n than those used in Figure 5 (1000 m and 0.2, respectively). For example, given the same K and R as in the hypothetical example where the residence time is 15.2 yrs, the residence time would be 30.4 yrs, if the island were 2000 m wide, or the porosity were 0.4.

SPATIAL VARIATION IN HYDRAULIC CONDUCTIVITY: BERMUDIAN- AND BAHAMIAN-TYPE ISLANDS

Increase of Hydraulic Conductivity with Age

Smith et al. (1976, Fig. 6.3) give a useful summary plot showing the geological variability of hydraulic conductivity of limestones (Fig. 6). The hydraulic conductivities shown in the figure were calculated by Smith et al. (1976) by assuming the pore system to be a bundle of straight, parallel tubes. As shown in the figure, the hydraulic conductivities of limestones (not counting the "marble") range from less than 10^{-3} m/day in "massive limestones" to more than 10^{6} m/day in cave systems -- more than 9 orders of magnitude. The important point for the present

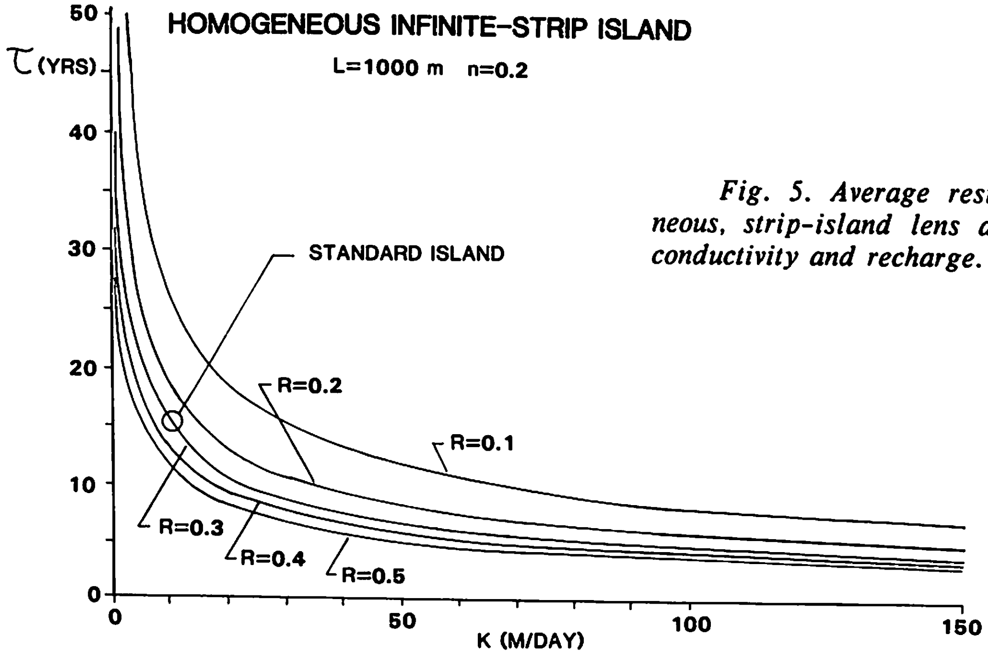


Fig. 5. Average residence time in homogeneous, strip-island lens as function of hydraulic conductivity and recharge.

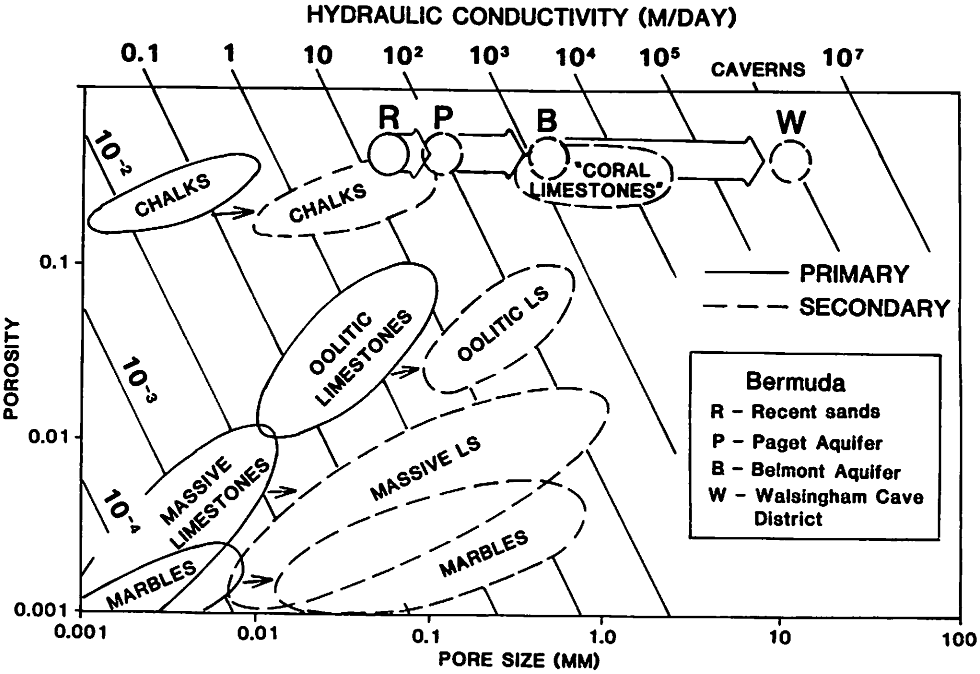


Fig. 6. Hydraulic conductivity of limestones as function of development of secondary porosity. The base figure is from Smith et al. (1976). The added data from Bermuda indicate the increase in hydraulic conductivity as a function of age due to meteoric diagenesis and karstification.

discussion is the orders-of-magnitude difference in hydraulic conductivity as secondary porosity is developed. According to the numbers of Smith et al. (1976), the hydraulic conductivity of oolitic limestones with primary porosity is in the range of 10^{-1} to 10^{1} m/day, and, with the development of secondary porosity ("fissured oolites," according to Smith et al., 1976), the hydraulic conductivity of the oolitic limestones is 1 to 2 orders of magnitude higher, 10^{1} to 10^{2} m/day. Young "coral limestones" are higher, 10^{3} to 10^{4} m/day, according to Smith et al. (1976), and truly cavernous limestones are some 2 orders of magnitude higher still.

The numbers calculated by Smith et al. (1976) are similar to those found from hydrogeologic studies in Bermuda. Representative Bermudian hydraulic conductivities are included in Figure 6. These numbers are obtained by a variety of methods that involve large-scale, in-situ stress tests, such as response to pumping and dampening of ocean tides (Vacher, 1978). As such, they are large-scale, bulk parameters, representing an average over a significant volume of rock. They are not comparable to the lower permeabilities determined on the scale of perm plugs, for example, which give only the permeability between the more important, secondary pores.

Typical Bermuda hydraulic conductivities are on the order of 10^{1} to 10^{3} m/day for the Pleistocene eolianites that occur in the region of the fresh-water resource. A fundamental fact is that the lower values occur in younger stratigraphic units. For example, the two principal hydrostratigraphic units are the Paget and Belmont aquifers (Vacher, 1978; Rowe, 1984). The Paget unit was deposited during the Sangamon Interglacial, 130 to 80 thousand years ago. The hydraulic conductivity of most of this unit is about 80 to 100 m/day. Younger portions of the Paget aquifer correspond to the Southampton Formation (Land et al., 1967; Vacher and Harmon, 1987) which was deposited during post-Substage 5e sea-level events, and in these areas the Paget aquifer appears to have a hydraulic conductivity of about 30 m/day. In contrast to the Paget, the Belmont aquifer has a hydraulic conductivity of about 1000 m/day; this heterogeneous unit includes the Belmont and Town Hill Formations (Vacher and Harmon, 1987), which were deposited in earlier, mid-Pleistocene interglacials (probably during Isotope Stages 7 to 11). At the low extreme of the range of hydraulic conductivity in Bermuda, the recent beach and dune sediment has a hydraulic conductivity of about 20 m/day, judging from its grain-size characteristics and absence of secondary porosity. At the high extreme of the range, there is the Walsingham cave district, which occurs in the oldest limestone formation exposed on the island. Water-table fluctuations in the caves (measured by Tom Iliffe, pers. comm., 1982) suggest a hydraulic conductivity comparable to the 10^6 and 10^7 m/day that Smith et al. (1976) estimated for "caverns" in their summary figure (Fig. 6).

All these Bermudian limestones consist of the same material (bioclastic calcarenite) and were deposited in the same environment (coastal dunes and associated beaches). The differences in hydraulic conductivity reflect only differences in age -- i.e., the extent of alteration by meteoric diagenesis and karstification. This pattern of increasing hydraulic conductivity with increasing age reflects the redistribution of porosity that occurs as the limestones are subjected to circulation of meteoric ground water through island lenses and associated mixing zones. This alteration in fresh-water lenses and mixing zones occurred discontinuously, as the limestones were exposed to phreatic processes only during the peaks of sea-level highstands; therefore, the correlation of larger hydraulic conductivity with age can also be viewed as a correlation of hydraulic conductivity with number of phreatic soakings. In any case, the transformation is from intergranular to cavernous porosity, and it is accompanied by a reduction in the ratio of surface area to pore volume within the pore system (although bulk porosity may actually be reduced; Land et al., 1967). The result is the observed trend: older limestones are sequentially more permeable, by orders of magnitude as shown in Figure 6.

Bermudian- and Bahamian-type Islands

Given the pattern of increasing hydraulic conductivity with increasing geologic age for limestones altered by coastal and island ground waters in the zone of eogenetic diagenesis (Choquette and Pray, 1970), we can define 3 patterns of hydraulic-conductivity variation in small limestone islands such as occur in Bermuda and the Bahamas:

1. Small islands composed of a single hydrostratigraphic unit. Small, in this case, is a relative term, meaning that the areal dimensions are sufficiently small that the interface does not intersect another hydrostratigraphic unit. Obvious examples include Bahamian oolitic cays such as Joulters and the Schooners studied by Halley and Harris (1979) and Budd (1984), respectively, and the many erosional outliers off the 5 main islands comprising Bermuda.

2. Larger islands in which there is a lateral succession of stratigraphic units. Larger islands of Bermuda are examples (Vacher, 1987). These islands are composed of a lateral succession of Pleistocene beach-dune complexes deposited during successive interglacials. Beach-dune complexes of more recent interglacials onlap the seaward margin of earlier-deposited, more diagenetically altered limestones. Older limestones border inshore water bodies (sounds and reaches off the North Lagoon), and younger limestones border the external shoreline. The overall pattern of cross-island sections, then, is one of lateral accretion of successively less-altered limestones, from large hydraulic conductivities along interior shorelines to smaller hydraulic conductivities along external shorelines. These islands with a lateral variation in hydraulic conductivity will be called "Bermudian-type islands."

3. Larger islands in which there has been vertical buildup rather than lateral accretion. In these islands, older, more permeable units completely underlie the less altered units that occur at the water table. According to the analysis by Cant and Weech (1986), larger islands of the northern Bahamas, (e.g., Grand Bahama and Abaco) are examples. The main hydrostratigraphic contact is the base of the Pleistocene Lucayan Formation. Cant and Weech (1986) also note the presence of discontinuity or subaerial-exposure surfaces within the Pleistocene section that correspond to abrupt downward increases in hydraulic conductivity. Such islands with low-K layers above high-K layers will be called "Bahamian-type islands."

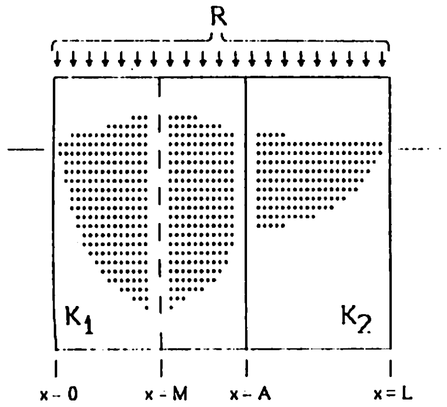
SHAPES OF LENSES IN BERMUDIAN- AND BAHAMIAN-TYPE ISLANDS

1. Small Cays

Small islands composed of a single uniform lithology (e.g., Joulters, Schooners Cays) are homogeneous islands. They contain the simple, symmetric lenses like that shown in Figure 1a. Equations of Figure 4 apply.

2. Bermudian-Type Islands

If the upper saturated zone consists of laterally adjoining sectors of different hydraulic conductivity, the fresh-water lens is asymmetric. Equations for the water table, depth to interface,



K_1, K_2 hydraulic conductivities   A = coordinate of geologic contact

Then:

(1) Divide: $$M = (1/2)\,\frac{K_1(L^2 - A^2) + K_2A^2}{K_1(L - A) + K_2A}$$

(2) Water table

(a) $0 \le x < A$ $$h^2 = \frac{R}{K_1(\alpha + 1)}(2Mx - x^2)$$

(b) $A \le x < L$ $$h^2 = \frac{R}{K_1(\alpha + 1)}(2MA - A^2) + \frac{R}{K_2(\alpha + 1)}[2M(x - A) - (x^2 - A^2)]$$

(3) Interface: $z = \alpha h$

(4) Fresh-water thickness: $H = (\alpha + 1)h$

Fig. 7. Dupuit-Ghyben-Herzberg equations for configuration of lens in a strip island with Bermudian-type variation in hydraulic conductivity. Parameters same as in Figure 4. Derivations are in Vacher (1988).

and thickness for the two-sector case (Vacher, 1988) are given in Figure 7.

Cross sections calculated from these equations are shown in Figure 8. The set of cross sections shows two ratios of hydraulic conductivities (K2/K1 at 10 and 100) and three positions of the geologic contact (at 0.25, 0.5, and 0.75 of the island width). The scales showing island width, depth to interface, and elevation of the water table in meters are based on an island where L=1000 m, R=0.3 m/yr, and K1=10 m/day. In addition to these scales, a dimensionless scale for the depth to the interface (z/zc) is also shown. This scale indicates the ratio of the depth of the interface (z) in the two-K lens to the depth of the interface at the midline (x=L/2) of the lens that would occur if the island were composed only of K1 (zc). The dimensionless cross sections (h/hc and z/zc vs x/L) apply to any lens with the same ratio of A/L and K2/K1 (i.e., they do not depend on L, R or K1; Vacher, 1988).

From Figure 8, it is obvious that the amount of asymmetry of the lens in a Bermudian-type island depends on the contrast in hydraulic conductivity between the different-K segments and the location of the contact. The thickest column of fresh ground water is in the lower-K unit, which acts as a relative dam to the shoreward flow of fresh ground water; the high-K unit acts as a relative drain. Accordingly, the groundwater divide occurs relatively close to the shoreline bordered by the lower-K rocks, and the majority of recharge exits at the higher-K shoreline.

The Bermuda example is shown in Figure 9.

3. Bahamian-Type Islands

If the upper saturated zone consists of horizontal layers of contrasting hydraulic conductivity, the effect of the refraction of the interface as it passes from one layer into the other will be a thinning of the lens from the geometry that would occur if there were no geologic contact. The thinning is reflected in a lower water table, as seen from the Ghyben-Herzberg Principle. Equations for the two-layer case (Vacher, 1988) are shown in Figure 10. Examples calculated from these equations are shown in Figure 11. The examples are analogous to those of Figure 8 in the K2/K1 ratios and the relative position of the geologic contact.

As in Figure 8, the scales in Figure 11 giving the depth to the interface in meters are based on an island with L=1000 m, R=0.3 m/yr, and K1=10 m/day. Also as in Figure 8, the dimensionless scales in Figure 11 reflect the comparison of interface depth in the two-layer case to the corresponding midline value in the comparison island with the same L and R, and a single K=K1. In the two-layer case, the depth below sea level of the geologic contact (b) is made dimensionless relative to zc; the three examples are where the geologic contact lies at 25, 50, and 75 percent of the depth of the center-line interface in the one-K comparison island. Then, with these conventions, the dimensionless cross sections apply to any Bahamian-type two-layer lenses with the same K2/K1 and b/zc, regardless of recharge, size of the island, and numerical value of the hydraulic conductivities (Vacher, 1988).

As shown in Figure 11, the amount of thin-

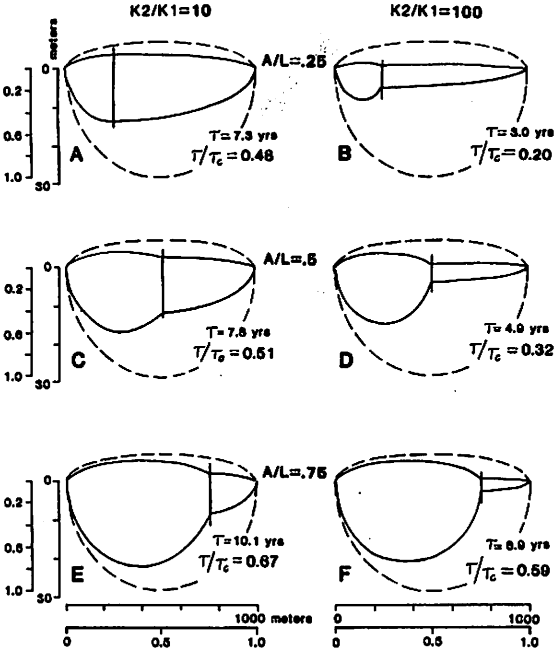


Fig. 8. Shape and average residence times in selected Bermudian-type lenses. Dashed lines indicate the comparison lens, which would be present if the island were composed only of the K1 unit.

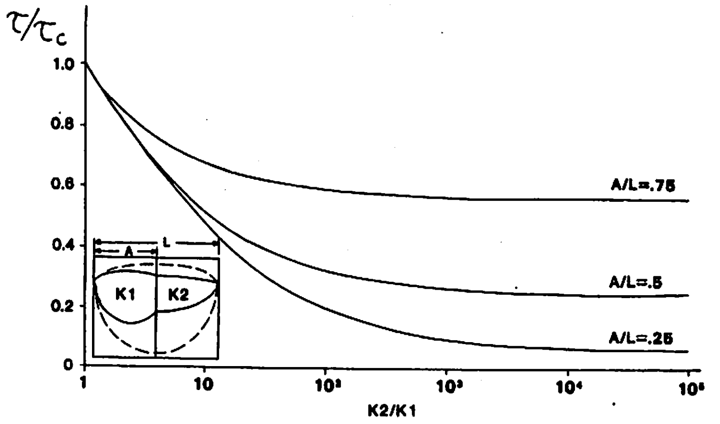


Fig. 10. Dupuit-Ghyben-Herzberg equations for configuration of lens in a strip island with Bermudian-type variation in hydraulic conductivity. Parameters same as in Figure 4. Derivations are in Vacher (1988).

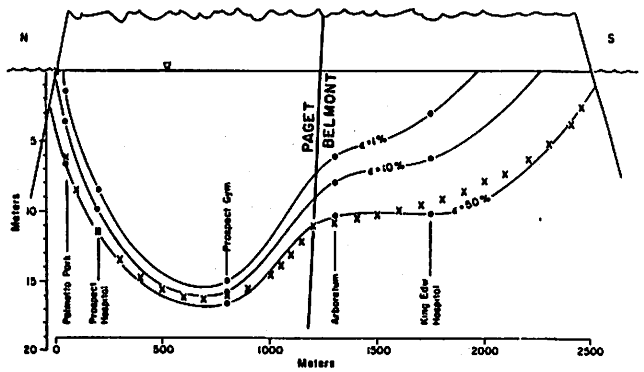
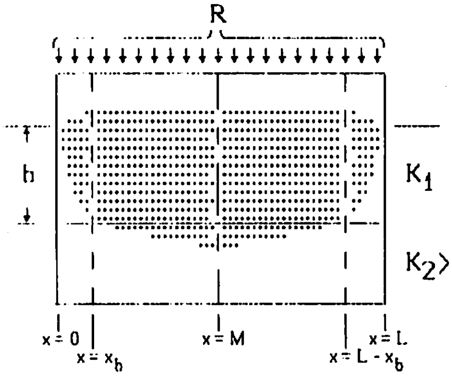


Fig. 9. Configuration of Devonshire lens Bermuda (Vacher, 1978). The 50%-seawater line indicates the position of the interface, which is at a depth 40 times the elevation of the water table (Rowe, 1984). The x's indicate the calculated position of the interface using Dupuit-Ghyben-Herzberg analysis with Bermudian parameters for recharge and hydraulic conductivities (Vacher, 1974, 1978).



K_1, K_2 hydraulic conductivities

b -- depth of geologic contact below sea level

x_b -- coordinate of intersection of interface and geologic contact

h_b = elevation of water table at $x = x_b$

Then:

(1) Divide: $M = L/2$

(2) Intersection coordinate from: $2Mx_b - x_b^2 = K_1(\alpha + 1)b^2/RA^2$

(3) Water table:

(a) $0 \leq x \leq x_b$ $\qquad h^2 = \dfrac{R}{K_1(\alpha + 1)}(Lx - x^2)$

(b) $x_b < x \leq M$

$$(K_1 + \alpha K_2)(h^2 - h_b^2) + 2(K_2 - K_1)b(h - h_b) = R[L(x - x_b)(x^2 - x_b^2)],$$

where $h_b = (R/K_1)(Lx_b - x_b^2)/(\alpha + 1)$

Fig. 11. Shape and average residence times in selected Bahamian-type lenses.

ning due to the deeper high-K unit is related to the contrast in hydraulic conductivities and the depth of the geologic contact. In the extreme (K2/K1 infinite), the lens would be truncated at the geologic contact, so fresh ground water would occur only in the uppermost layer.

Such cases of strong geologic control by deeper, large-K units are described in the summary of Bahamian lenses by Cant and Weech (1986). For example:

> *"The Pleistocene formation, named the Lucayan Limestone..., is the main freshwater aquifer of the Bahamas.... No older formations are known to be able to prevent freshwater from mixing with saline water. Because of this, the base of the Lucayan Limestone represents the maximum thickness to which the freshwater lens can develop....*
>
> *"...It is the shallow depth of the Lucayan Limestone in the Little Bahama Bank that effectively limits the freshwater lens thickness in Grand Bahama and Abaco islands. These islands are larger than Eleuthera and have greater rainfall, but the freshwater lenses are thinner than on Eleuthera; this is due simply to the fact that the Lucayan Limestone extends down to about 40 m in Eleuthera as compared to 21 m below sea level in the Little Bahama Bank."*

Cant and Weech (1986, p. 341) also describe the similar effects of changes in hydraulic conductivity within the Lucayan:

> *Throughout the Lucyan Limestone there are discontinuity surfaces that are interpreted as horizons of subaerial exposure.... Rapid changes in rock character can occur across a discontinuity surface, and in particular at the major one that occurs at a depth ranging from 8 to 10 m in the Central Bahamas. This particular change demarcates a significant change in rock permeability downwards, and it acts as a common barrier to the development of freshwater lenses in the smaller islands. This is a well known constraint on the island of New Providence...*

From the cross sections in Figure 11 for K2/K1=100, it is easily seen how the base of the Lucayan Limestone can exert the kind of geologic control that Cant and Weech (1986) document. With two orders of magnitude contrast in hydraulic conductivity, the critical control on the thickness of the lens would be the depth to the contact.

Comments

The classification of Bermudian-type and Bahamian-type hydraulic-conductivity variations and lens geometries is intended to characterize endmembers only, and it is somewhat arbitrary. From the descriptions by Cant and Weech (1986), Bermudian-type hydraulic-conductivity variations and lenses are present locally in the Bahamas, and there are also lens geometries that do not fit either pattern. For example, Cant and Weech (1986, p. 342) note the following Bermudian-type relation between the lower-K Holocene sands and the higher-K Lucayan:

> *"In addition to the Lucayan Limestone, freshwater can also occur in Holocene sands. Very large lenses are known to occur in parts of Eleuthera, Abaco, Cat Island, Exuma, and in many other islands where such sand bodies occur in coastal areas. Quite often freshwater will occur in a sand where it would not occur in a rock formation, and this is because permeabilities are so low in the sand.... On many of the long thin islands of the central Bahamas, freshwater only occurs where there are sands that can effectively reduce seepage losses to sea."*

Figure 8B shows the kind of geometry that seems to be implied by the descriptions of Cant and Weech (1986) for islands where a package of low-K Holocene sand abuts against the highly permeable limestone bedrock that comprises the main part of the island.

Cant and Weech (1986) also indicate that in the southern, drier Bahamas, there is sufficiently low recharge that the thin lenses are effectively "broken up" by hypersaline ponds which, from their descriptions, act as evaporation sinks. The resulting lens configuration then would be the result of lateral variations in recharge (with negative R at the sinks), rather than a control by the distribution of hydraulic conductivity. The descriptions from Cant and Weech (1986) that we quote above to illustrate lenses with a Bahamian-type variation in hydraulic conductivity are said by Cant and Weech (1986) to be characteristic in the northern Bahamas which receive more than some 115 cm/yr of rainfall.

It should be noted also that a Bahamian-type variation in hydraulic conductivity is present in Bermuda as well as in the Bahamas. Deep drilling beneath the Bermudian lenses reveals cavernous limestone near the contact with the underlying volcanic seamount. From the limited data available, tidal fluctuations in head are significantly larger in these deeper units than are the fluctuations of the water table, meaning there is a significantly larger hydraulic conductivity of these deeper limestones. These deeper limestones, however, are well below the depth reached by the interface in the Paget and Belmont aquifers, so these deeper layers play no role in the configuration of the lens. This would not be the case at a lower stand of sea level, however, when the exposed Bermuda would have the dimensions similar to a present-day Bahamian island.

RESIDENCE TIME IN BERMUDIAN- AND BAHAMIAN-TYPE ISLANDS

The residence time for each of the patterns of hydraulic conductivity variation can be found from the analytical solutions for the across-island variation of the thickness of the lens. The volume of the lens is found from numerical integration of Hdx using the equations in Figures 7 and 10.

Bermudian-Type Islands

Calculated results for the six cases of Figure 8 are shown in that figure. For example, the average residence time (τ) in a two-sector lens with K1=10 m/day, K2=100 m/day, R=0.3 m/yr, n=0.2, and L=1000 m is 7.8 yrs, if the geologic contact is at A=500 m (Fig. 8C). The residence time (τ_C) of the comparison homogeneous island (K=10 m/day, R=0.3 m/yr, L=1000 m, and n=0.2) is 15.2 yrs, from equation 4. Therefore the dimensionless residence time (τ/τ_C) for this two-layer island is 0.51. This dimensionless residence time holds for any two-layer island with the same K2/K1 and A/L, independent of size, recharge, porosity and hydraulic conductivity. That is, the effect of the high-K sector in a Bermudian-type island is a reduction of the residence time by 49 percent, if the hydraulic conductivity of the higher-K sector were an order of magnitude larger and the sector occupied 50 percent of the cross-island width of the lens.

Dimensionless residence time is shown as a function of K2/K1 and A/L in Figure 12. Obviously, the amount of reduction of residence time is related to the contrast in hydraulic conductivity and the percentage of the island that is made up of the higher-K rock.

Bahamian-Type Islands

Calculated average residence times for the six Bahamian-type islands of Figure 11 are shown in that figure. As in Figure 8, the results are given both in terms of years for the particular cases calculated and dimensionless residence times formed by comparison with the homogeneous lens with K=K1. These dimensionless residence times are the same in all lenses with the same K2/K1 and b/zc, where the latter is the ratio of the depth to the geologic contact to the depth of the center-line interface (x=L/2) in the comparison island. For example (Fig. 11 C), the effect of the buried high-K layer would be to reduce the residence time by 19 percent if there were an order-of-magnitude contrast in hydraulic conductivity, and the geologic contact were to occur half way to the deepest interface that would be present if the deeper layer were not present.

Dimensionless residence time for Bahamian-type islands is shown as a function of K2/K1 and b/zc in Figure 13. As in the Bermudian-type island, the amount of reduction of residence time varies directly with the permeability contrast and inversely with the percentage of the comparison-island lens that is occupied by the lower-K unit. If the depth to the geologic contact is relatively large, so that the deeper unit "chops off" only the lower part of the homogeneous-island lens, then the reduction in residence time, obviously, is relatively small.

Comments

Other examples than those given here (Figs. 8 and 11) can be easily calculated from the properties of dimensionless residence time and the graphs of dimensionless residence time vs ratios of hydraulic conductivity (Figs. 12 and 13). One need not get into the equations and numerical integration; one only needs to recall that

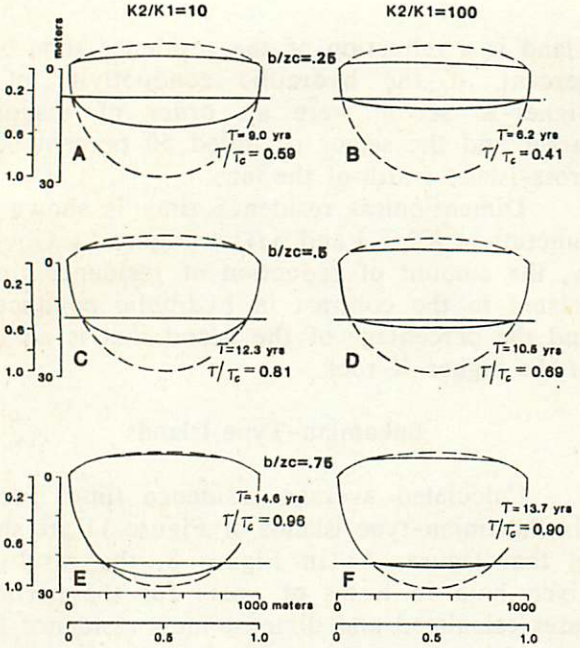


Fig. 12. Dimensionless average residence time in a two-sector, strip-island lens with a Bermudian-type hydraulic-conductivity variation.

average residence time is 15.2 yrs in the standard island of Fig. 5 (a homogeneous island with K=10 m/day, R=0.3 m/yr, n=0.2, and L=1000 m). For Bahamian-type islands, it is useful to recall that the deepest interface is 28.3 m for this same island. To illustrate the technique, consider the following problem set of two "unknowns."

a. Suppose the island is 3000 m wide and consists of permeable Pleistocene limestones onlapped by a package of Holocene sand. Suppose the Holocene unit is 1000 m wide, and pump tests and/or tide studies indicate K=2000 m/day in the Pleistocene and K=40 m/day in the Holocene. Suppose that it is also known that recharge is 0.2 m/yr and the porosity of both units is 0.25.

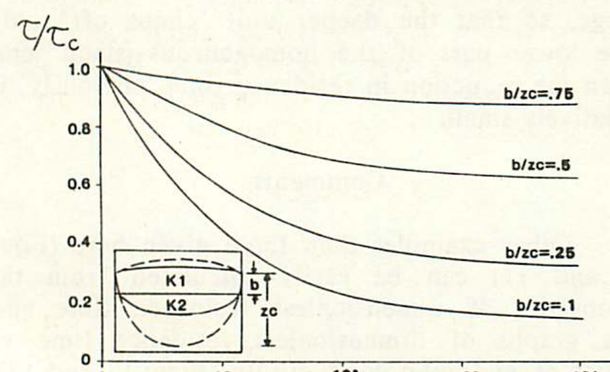


Fig. 13. Dimensionless average residence time in a two layer, strip-island lens with a Bahamian-type hydraulic-conductivity variation.

Then, remembering the effect of the variables in equation 4, the residence time of the comparison island consisting only of Holocene sand can be found from the ratios of the controlling variables:

$$\tau_C/15.2 =$$
$$(3000/1000)(0.25/0.2)(0.2/0.3)^{-1/2}(40/10)^{-1/2}$$

Then τ_C=32.9 yrs. Next, the critical ratios for our problem island (K2/K1=50 and A/L=.33) are used with the graphs of Figure 12 to estimate the dimensionless residence time for the problem island (approximately 0.3, by interpolation). This result, with τ_C, indicates that the average residence time in the problem lens is about 10 yrs.

b. Suppose the island is 8000 m across and consists of Pleistocene rocks (K=500 m/day) overlying older limestones (K=80,000 m/day). Suppose the contact is horizontal at 15 m below sea level, that R=0.4 m/yr, and n of both units is 0.3. Then, by the same technique as in the preceding example, the residence time of the comparison island (consisting only of Pleistocene rocks) is 22.3 yrs. In order to read the dimensionless residence time for this island from Figure 13, one needs the permeability ratio (160) and b/zc. The latter is found by remembering the effect of the controlling variables in equation 1; that is,

$$zc/28.3=(0.4/0.3)^{1/2}(500/10)^{-1/2}(8000/1000) \ ,$$

Then zc=37 m, and b/zc=0.41. With these ratios, the dimensionless residence time for the problem island is interpolated on Figure 13 (approximately .6). The average residence time in the problem island is 8.5 years.

DISCUSSION

1. The residence time discussed in this paper is the overall average. Considering the individual parcels of water that pass through the lens, there is a wide range of residence times depending on the travel lengths of the various flow paths (Fig. 2). Residence time as a function of entry point can be found either by flow-net analysis or by a mathematical technique to find the stream function in the lens (Vacher et al., in review). Such work indicates that some 95% of the water that passes through the lens resides in the lens for less than about 6 times the overall

average residence time. Therefore, given the results of the preceding problem as an example, it is probably safe to say that practically all the fresh ground water passes through that island in less than 75 yrs, while most of it passes through in less than 10 yrs.

2. Although we have used the term "Bahamian-type lens" and it seems appropriate from the descriptions by Cant and Weech (1986), the Bahamian-type variation in hydraulic conductivity, where less-permeable limestones overlie more-permeable limestones, is certainly not limited to the Bahamas. In fact, it is widespread, and perhaps a characteristic of islands composed of relatively young limestones. The presence of buried highly permeable units in Enewetok Atoll is well known (Wheatcraft and Buddemeier, 1981), and they play an important role in the propagation of the tidal signal there. However, these units are below the lens, because the emergent atoll island is very small. The same is true in other atolls, such as Pingelap (Ayers and Vacher, 1986) and Kwajalein (Hunt and Peterson, 1980). In Pingelap, the shape of the lens is controlled partly by a lateral variation in hydraulic conductivity which is a reflection of grain-size differences between reef-bordering and lagoon-bordering sediments of the Holocene aquifer (and partly by the presence of very low-permeability reef plate near the reef). The atoll-island lens, therefore, is asymmetric as in Bermuda; however, it is less "bulbous" than in Bermuda, because the change in hydraulic conductivity is gradual rather than abrupt. But, in all these islands -- the three atolls and Bermuda as well -- a Bahamian-type downward increase in hydraulic conductivity is present or suspected from the tidal data. Similarly, in Big Pine Key (Stewart, 1988), a Bahamian-type change in hydraulic conductivity is present where the Miami Oolite overlies the Key Largo Limestone, and that contact controls the thickness of the lens.

To the extent that the configuration of the lens in limestone islands can be generalized, therefore, it appears that the size of the island is the principal control on the shape of the lens. Higher-permeability units tend to occur at depth below the sediments and rocks that occur at the water table. The cross-island geometry of the lens, then, depends on whether the size of the island (and, to a lesser extent, the recharge) is large enough to cause the interface to encounter these higher-permeability rocks. The larger Bahamian islands and Big Pine Key are large enough, so there is the kind of geologic control documented by Cant and Weech (1986) in the Bahamas. In smaller islands, where the lens's shallow interface is unlikely to encounter buried higher-permeability rocks, the cross-island variation in lens geometry depends on whether there is a lateral variation in hydraulic conductivity. In Bermuda, such a variation is present in the larger islands because of the lateral accretion pattern of the sedimentary buildup of that island. In atoll islands, a lateral variation in hydraulic conductivity is present because of the spread of a wide range of grain sizes in the storm-deposited reef debris. In both cases, the resulting lenses are asymmetric -- with thickest fresh-water columns near the young-rock shoreline in Bermuda and near the lagoon in atoll islands. In small islands that are composed of a single-age sediment or rock consisting of uniform grain sizes, the simple symmetric lens occurs. The small Bahamian cays of Joulters and the Schooners are examples.

3. The equations for a homogeneous island imply that the thickness of the lens is a few percent of the width of the island. This rule of thumb is limited to the small, young-limestone islands where homogeneous hydraulic conductivities are present. In the larger young-limestone islands, higher hydraulic conductivities are apt to be encountered by the interface, and the lenses are even thinner. This is true regardless of whether Bermudian-type or Bahamian-type variations in hydraulic conductivity are present; in both cases, the larger hydraulic conductivities lead to a smaller overall R/K ratio than the 10^{-6} that is consistent with the thickness/width ratio of about 1 percent. Given the widespread occurrence of high-permeability units that occur in the subsurface of present-day, isolated carbonate islands such as the Bahamas, Bermuda, and the atolls, it is difficult to see how Ghyben-Herzberg lenses, and associated diagenesis, can extend to below a few tens of meters below sea level, unless significant lower-permeability, presumably clastic units, are also present.

CONCLUSIONS

The average residence time of fresh ground water in an island lens depends on the volume of ground water in the lens. This volume is fundamentally related to the hydraulic conductivity of the sediments and rocks containing the lens. The effect of Bermudian-type high-permeability sectors and Bahamian-type high-permeability buried layers is to thin the lens and thereby reduce the residence time. These average residence times can be calculated from Dupuit-Ghyben-Herzberg analysis given knowledge of the recharge, location of geologic contacts, and magnitude of the hydraulic conductivities. If the island is an infinite strip and recharge is uniform, the shape of the lens and the average residence time of fresh ground water in it can be calculated from the equations and graphs of this paper.

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