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ALGEBRAIC MODEL TO PREDICT THE FRESHWATER LENS THICKNESS OF AN ATOLL ISLAND

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ABSTRACT

Several published analytical and empirical models have been proposed to estimate the thickness of the freshwater lens on oceanic islands. These models, however, have no provision for certain geologic features unique to the atoll island subsurface, and tend to overestimate the thickness of the freshwater lens when applied to atoll islands. In this paper an algebraic model is presented that relates the maximum thickness of the atoll islands freshwater lens to environmental factors outlined in the conceptual model presented by Ayers and Vacher (1986). These factors are the island width, the annual recharge, the Holocene sediment hydraulic conductivity, the Thurber Discontinuity depth, and the reef flat plate. The Thurber Discontinuity depth is the defining feature of the algebraic model, and provides for the accuracy of the model. The algebraic model produces results consistent with observed lens thicknesses on atoll islands across the Pacific and Indian Ocean basins, and may become a valuable tool to atoll island water resource managers.

INTRODUCTION

During times of water stress the residents of atoll islands often turn to groundwater to meet their domestic needs (Cox, 1951; MacCracken et al., 2007). However, hydrologic data on the quantity of groundwater is scarce for atoll islands due to their remote location and low population. Several analytical or empirical equations have been established in order to provide estimates of hydraulic head and freshwater lens thickness of circular and strip oceanic islands. Fetter (1972) and Chapman (1985) developed analytical solutions for determining the depth of the seawater/freshwater interface, defined by a chloride (Cl^-) concentration of 500 mg L^{-1} (Lloyd et al., 1980), beneath both strip and circular oceanic islands, taking into account the width of the island, average annual recharge rate, and aquifer hydraulic conductivity. Using the observed lens thickness from eight small coral islands, Oberdorfer and Budde-meier (1988) published an empirical relationship between the ratio of the lens thickness and the annual rainfall, and the logarithm of the island width.

These models, however, have limited success when applied to atoll islands (Falkland 1994). Atolls islands possess dual aquifers (Ayers and Vacher, 1986), in which a fine-deposit Holocene aquifer is superimposed upon a more conductive karstic Pleistocene aquifer (Thurber et al., 1965) (Figure 1). The contact is an unconformity located 15-25 m below sea level (Wheatcraft and Buddemeier, 1981; Hamlin and Anthony, 1987; Dickinson, 2004).

Hydraulic conductivity of the Holocene aquifer depends on the position of the island relative to the prevailing winds (Anthony, 1997; Spennemann, 2006) and is estimated to be one to two orders of magnitude less than that of the Pleistocene aquifer (Hunt and Peterson, 1980; Woodroffe and Falkland, 1997). The higher conductivity of the underlying Pleistocene aquifer truncates the freshwater lens at or near the contact of the two units (Falkland, 1994), thus limiting the thickness of the freshwater lens. Another distinctive feature of atoll aquifers is a reef flat plate extending inward from the ocean side of the island (Figure 1). Where present, it partially confines the Holocene aquifer, forcing discharge upward through fractures in the plate or laterally at the reef margin (Buddemeier and Holladay, 1977; Ayers and Vacher, 1986). The accuracy of a model applied to atoll island hydrogeology depends upon the inclusion of these geologic features. In this paper we present a simple two-dimensional algebraic model, based on results from numerical simulations (Bailey, 2008), that takes into account these unique features of atoll island hydrogeology.

ALGEBRAIC MODEL

Development of Algebraic Model

The development of the algebraic model was based on results from numerical simulations of atoll island groundwater flow (Bailey, 2008). Baseline sensitivity test series using generic numerical models of atoll islands were run to quantify

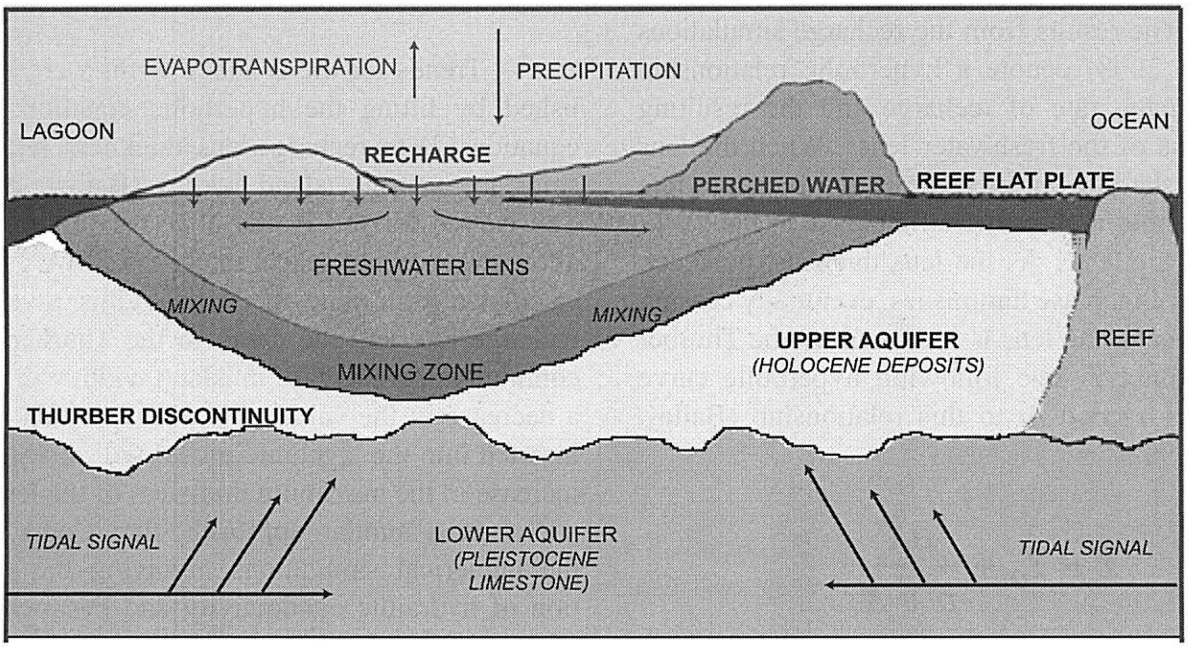

Figure 1. Conceptual model of the hydrogeology of an atoll island, after Ayers and Vacher (1986)

the influence of recharge, island width, hydraulic conductivity, Thurber Discontinuity depth, and the presence of the reef flat plate on the thickness of the lens at the groundwater divide, the location of maximum lens thickness (Figure 2).

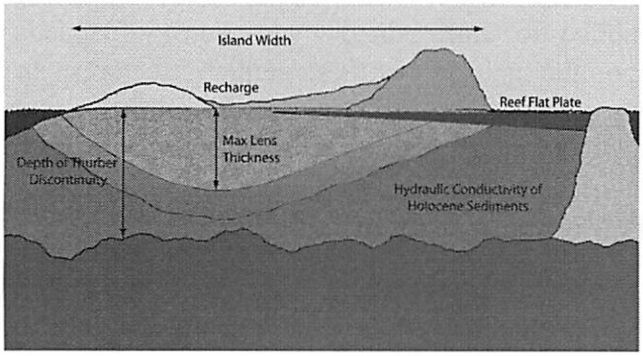

Figure 2. Factors influencing the thickness of the freshwater lens on atoll islands

Simulation results indicated that increasing the annual recharge rate, increasing the island width, including the presence of the reef flat plate, and decreasing the hydraulic conductivity of the Holocene sediments all resulted in a thickening of the freshwater lens.

The results from the recharge simulations (Bailey, 2008) denote a hyperbolic relationship between the rate of recharge and the resulting thickness of the freshwater lens. When the lens is very shallow, an increase in the recharge rate brings about a proportional increase in the thickness of the lens. As the lens thickens, however, the rate of increase diminishes, eventually ceasing altogether as the lens is truncated by the Thurber Discontinuity. The following hyperbolic curve provides a good fit to this relationship (Bailey, 2008):

(1)
$$Y = Y_o + \frac{aX}{b + X}$$

where:

X= Input to the system (recharge to the freshwater lens)
Y = Variable measured (thickness of the lens), and YO, a, and b are fitting terms.

By observing that the limiting value of the hyperbolic relationship is the depth to the Thurber Disconti-nuity and by replacing the parameters in equation (1) with terms common to hydrological studies, the following relationship is produced (Bailey 2008):

(2)
$$Z_{MAX} = \left[Y + \frac{(Z_{TD} - Y)R}{B + R} \right] (K)(C)$$

where:

ZMAX= Maximum thickness of the lens [m]
R= Recharge to the freshwater lens [m yr^{-1}]
ZTD= Depth to the Thurber Discontinuity [m]
Y, B= Constants, dependent on island width [-]

Trends for the B and Y terms were established by fitting the hyperbolic equation from equation (2) to recharge/lens thickness relationships for various island widths (Bailey, 2008). The curves for the B and Y terms (Figure 3A) allow the maximum lens thickness ZMAX to be calculated for a range of island widths, given the recharge rate and the depth to the Thurber Discontinuity. An increase in island width will yield a decrease in the value of B and Y, which when inserted into the algebraic model will produce an increase in the maximum thickness of the lens.

In a similar approach, the results from the numerical simulations involving the variation of hydraulic conductivity and the inclusion of the reef flat plate were used to create values for hydraulic conductivity and reef flat plate (confining) parameters (Bailey, 2008). These parameters were included in equation (3):

(3)

$$Z_{MAX} = \left[Y + \frac{(Z_{TD} - Y)R}{B + R} \right] (K)(C)$$

where:

K = Hydraulic conductivity factor [-]

C = Confining factor [-]

With the rest of the terms as defined previously. The value of the hydraulic conductivity parameter is determined from the curves in Figure 3B. An increase in the hydraulic conductivity yields a decrease in the hydraulic conductivity parameter value, which when included in the algebraic model will yield a de-crease in the maximum thickness of the lens. A decrease in the hydraulic conductivity will yield the opposite result. The effect of the hydraulic conductivity gives way to the effect of the Thurber Discontinuity for large islands. The value of the confining parameter is determined from Figure 3C. If the reef flat plate is not present, however, the parameter receives a value of 1.

The algebraic model is thus composed of two distinct parts: a hyperbolic curve that determines the thickness of the lens according to the recharge rate, island width, and the Thurber Discontinuity depth; and the inclusion of hydraulic conductivity and reef flat plate factors which either dampen or enhance the thickness provided by the hyperbolic curve. The parameters in the atoll island subsurface setting are portrayed in Figure 4.

Accuracy of Algebraic Model

The accuracy of the algebraic model, and hence its usefulness when applied to atoll island hydrology, is depicted in Figure 5, which com-

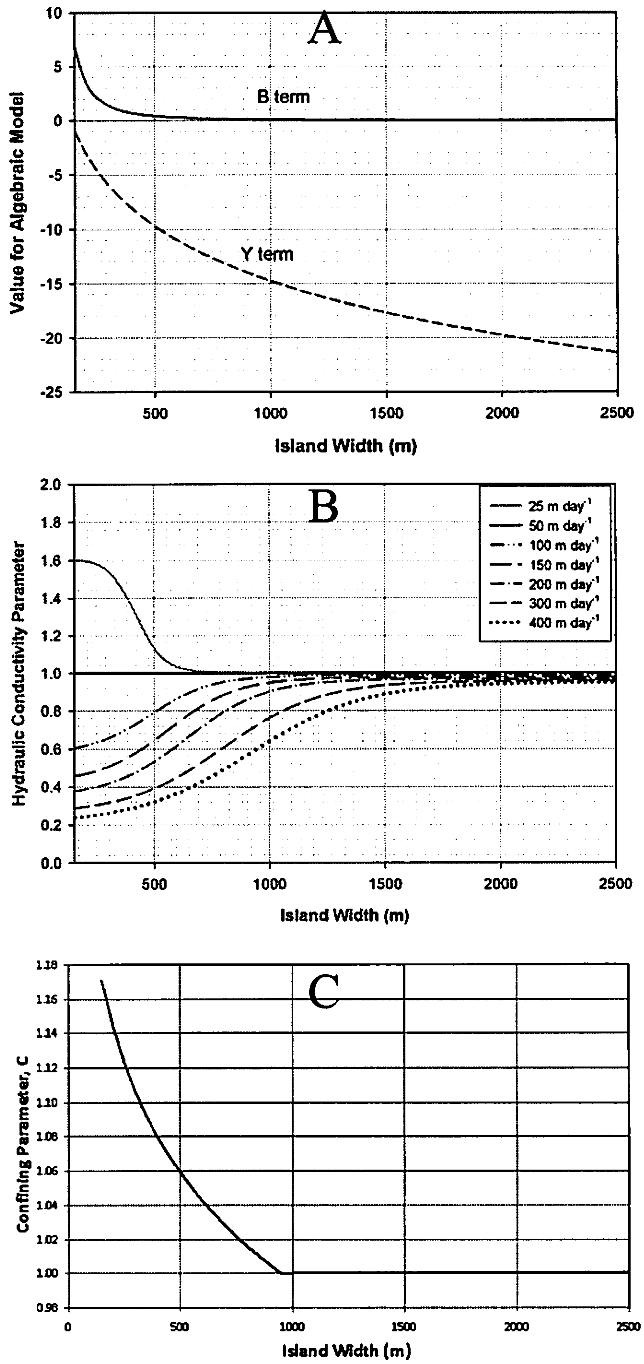

Figure 3. Curves for determining the values used in the algebraic model. (A) curves for [B] and [Y] parameters, based on islands width; (B) curve for the[K] parameter, based on hydraulic conductivity of the Holocene deposits; and (C) curve for the [C] parameter, based on the presence of the reef flat plate. The [C] parameter graph is used when the reef flat plate is present.

pares the results from the algebraic model to those from the analytical models (Fetter, 1972; Chapman, 1985) and the empirical model of Oberdorfer and Buddemeier (1988). The comparison uses observed lens thicknesses from leeward islands on atolls across the Pacific and Indian Oceans (Bailey, 2008). The hydraulic conductivity used in the analytical models of Fetter (1972) and Chapman (1985) and the algebraic model was 50 m day^{-1}. This value was shown by Bailey (2008) to approximate the hydraulic conductivity, based on agreements between observations and numerical simulation results. The results of the algebraic model are shown with two curves: the dashed curve used a Thurber Discontinuity depth of 15 m, while the solid curve used a depth of 20 m. The comparison between the models in Figure 5 accentuates the role of the Thurber Discontinuity in limiting the thickness of the lens, as well as the inability of the analytical and empirical models to accurately estimate this thickness.

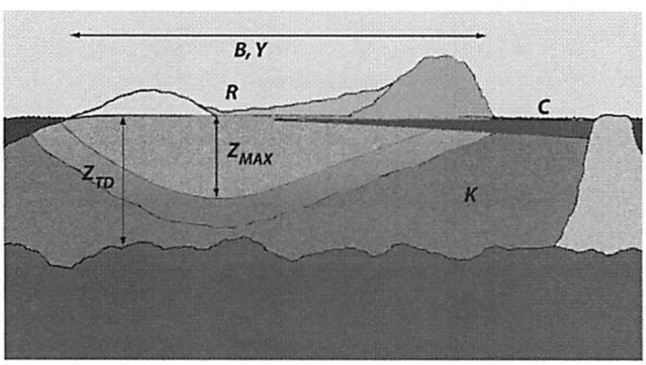

Figure 4. Physical representation of the parameters included in the algebraic model. ZTD = depth to the Thurber Discontinuity (m), ZMAX = maximum thickness of the freshwater lens (m), K = parameter representing the influence of the hydraulic conductivity of the Holocene sediments, R = annual recharge rate (m yr^{-1}), C = confining parameter, B, Y = parameters representing the influence of the width of the island

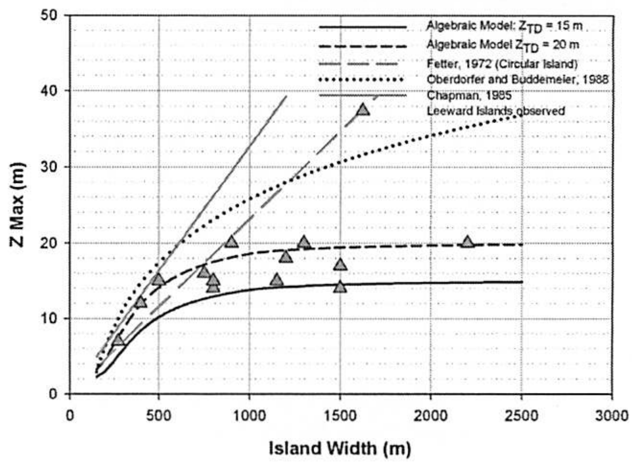

Figure 5. Observed lens depths on atoll islands of varying width based on results from the algebraic model presented herein and the analytical and empirical models. Values and references for leeward island observations can be found in Bailey (2008).

The Algebraic Model as a Tool

The algebraic model can be applied in one of two ways. First, the curves for the island width (B and Y), hydraulic conductivity (K), and reef flat plate (C) parameters (Figure 3) can be used to determine the parameter values. Accompanied by estimates of the annual recharge rate (m yr-1) and the depth to the Thurber Discontinuity (m), ZMAX can be calculated. A second way is a set of recharge curves (Figure 6). These curves were created using a Thurber Discontinuity depth of 17.5 m and hydraulic conductivity values consistent with leeward and windward islands (Bailey, 2008). They are expected to become a valuable tool for those managing the water resources of atoll islands.

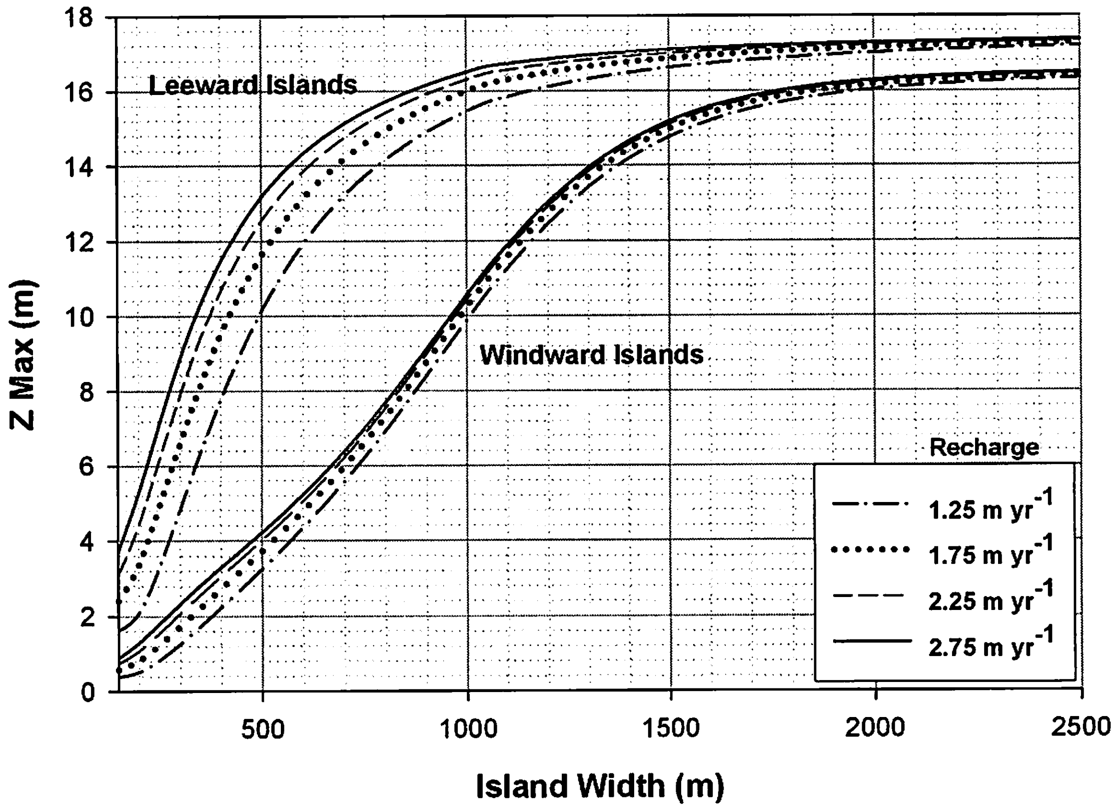

Figure 6. Curves used to estimate the thickness of the freshwater lens for atoll islands

CONCLUSIONS

The development of an algebraic model for estimating the freshwater lens thickness on atoll islands yields the following conclusions:

- The inclusion of the Thurber Discontinuity depth in a model estimating the freshwater lens thickness of atoll islands is necessary for accurate results.
- The algebraic model accurately estimates the freshwater lens thickness of leeward atoll islands.
- The algebraic model can be used in various forms, each of which may become a valuable tool to water resource managers of atoll islands.
- Caution is always needed when using simple models, such as the algebraic models. Unique geo-logic conditions may exist on an island which may not concur with the assumptions of the algebraic model.

The algebraic model presented in this paper is intended to be a tool used during the overall water re-sources analysis of atoll islands. Accurate predictions of the thickness of the lens, as provided by the algebraic model, will accentuate the fragility of the lens and promote the development of other sources of fresh water for atoll island communities.

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REFERENCES

Anthony, S.S. 1997. Hydrogeology of selected islands of the Federated States of Micronesia. in Geology and Hydrogeology of Carbonate Islands. Developments in Sedimentology 54, ed. H.L. Vacher and T. Quinn, 693-706.

Ayers, J.F., and Vacher, H.L. 1986. Hydrogeology of an atoll island: A conceptual model from detailed study of a Micronesian example. Ground Water 24, no. 2: 2-15.

Bailey, R.T. 2008. Groundwater resources of Micronesian atoll islands: observations, modeling, and management. M.S. thesis, College of Natural and Applied Sciences, University of Guam.

Buddemeier, R.W. and Holladay, G. 1977. Atoll hydrology: island groundwater characteristics and their relationship to diagenesis. In Proceedings; Third International Coral Reef Symposium, vol. 2,167-173.

Chapman, T.G. 1985. The use of water balances for water resource estimation with special reference to small islands. Bulletin No. 4. Pacific Regional Team. Australian Development Assistance Bureau, Canberra, Australia.

Cox, D.C. 1951. The hydrology of Arno Atoll, Marshall Islands. Atoll Research Bulletin no. 8: 1-29.

Dickenson, W.R. 2004. Impacts of eustasy and hydro-isostasy on the evolution and landforms of Pacific atolls. Palaeogeography, Palaeoclimatology, Palaeoecology 213, 251-259.

Falkland, A.C. 1994. Climate, hydrology and water resources of the Cocos (Keeling) Islands. Atoll Re-search Bulletin no. 400, 1-23.

Fetter, C.W. 1972. Position of the saline water interface beneath oceanic islands. Water Resources Re-seartch 8, 1307-1315.

Hamlin, S.N. and Anthony, S.S. 1987. Groundwater resources of the Laura area, Majuro Atoll, Mar-shall Islands. USGS Water Resources Investigation Report 87-4047, 69 p.

Hunt, C.D., Jr. and Peterson, F.L., 1980. Groundwater resources of Kwajalein Island, Marshall Islands. Technical Report No. 126, 91 p Water Resources Research Center, University of Hawaii

Lloyd, J.W., J.C. Miles, G.R. Chessman, and S.F. Bugg. 1980. A ground water resources study of a Pa-cific Ocean atoll – Tarawa, Gilbert Islands. Water Res. Bull. 16, no. 4: 646-653.

MacCracken, R.S., Jenson, J.W., Heitz, L.F., Rubinstein, D.H., Mylroie, J.E. 2007

Water Resources Analysis of Fais Island, Federated States of Micronesia. Technical Report No. 111. Water and Environmental Research Institute of the Western Pacific, University of Guam.

Oberdorfer, J.A. and Buddemeier, R.W. 1988. Climate change: effects on reef island resources. In Proceedings; Sixth International Coral Reef Symposium, vol. 3, 523-527.

Spennemann, D. 2006. Non-traditional settlement patterns and typhoon hazard on contemporary Majuro Atoll, Republic of the Marshall Islands. Transforming Cultures eJournal 1 no. 2: 44-63.

Stephenson, R.A. 1984. A Comparison of freshwater use customs on Ultihi Atoll with those of selected other Micronesia islands. Technical Report No. 51, 27 p. Water and Energy Research Institute, Univer-sity of Guam.

Thurber, D.L., Broecker, W.S., Blanchard, R.L. and Potratz, H.A. 1965. Uranium-series ages of Pacific atoll coral. Science 149, 55-58.

Wheatcraft, S.W. and Buddemeier, R.W. 1981. Atoll island hydrology. Ground Water 19, no. 3: 311-320.

Woodroffe, C.D., Falkland, A.C. 1997. Geology and hydrogeology of the Cocos (Keeling) Islands. in Geology and Hydrogeology of Carbonate Islands. Developments in Sedimentology 54, ed. H.L. Vacher and T. Quinn, 885-908.